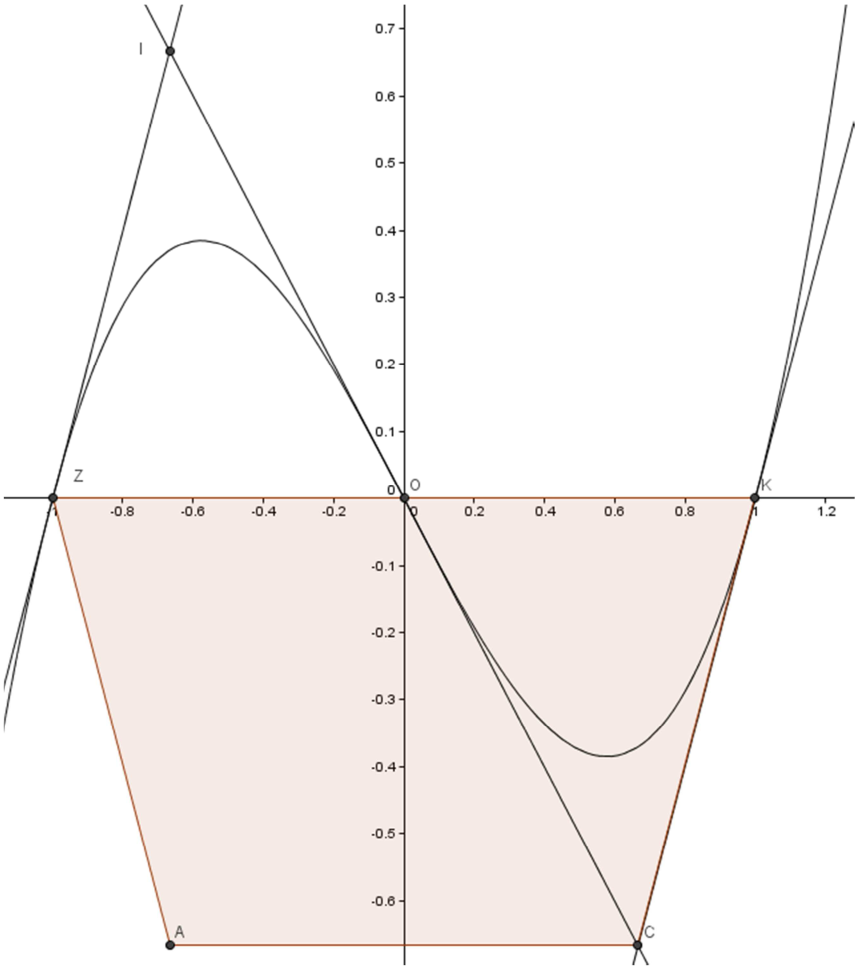

a: zu zeigen: f(z) = - f(-z)

r. S. f(-z) = $-[(-z)^3 - (-z)] = -[-z^3 + z] = z^3 - z = f(z)$ q.e.d.

b: Nullstellen: $f(x) = x^3 - x = x\,(x^2 - 1) = 0 \Rightarrow$

I $x_O = 0$

$\vee$

II $x^2 - 1 = (x - 1)(x + 1) = 0 \quad \Rightarrow x_Z = -1 \vee x_K = 1$

c: Tangentengleichungen mit $f'(x) = 3x^2 - 1$

K: $f'(1) = 3\text{-}1 = 2$, also t_K: $y = 2\,(x - 1) + 0 = 2x - 2$

O: $f'(0) = 0\text{-}1 = -1$, also t_O: $y = -1\,(x - 0) + 0 = -x$

Z: $f'(-1) = 3\text{-}1 = 2$, also t_Z: $y = 2\,(x+1) + 0 = 2x + 2$

Schnittpunkte der Tangenten

$t_Z \cap t_O = \{ I \}$

Schnittpunktgleichung: $t_Z(x) = t_O(x)$

$\Leftrightarrow \quad 2x + 2 = -x \mid + x - 2$

$\Leftrightarrow \quad 3x = -2 \mid *\frac{1}{2}$

$\Leftrightarrow \quad x_I = -\frac{2}{3}$

also I($-\frac{2}{3}/\frac{2}{3}$)

und damit gilt $\overline{OI} = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$

$$\overline{ZI} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}} = \frac{1}{3}\sqrt{5}$$

und wegen der Symmetrie: $\overline{ZICK} = 2 * \left(\frac{2}{3}\sqrt{2} + \frac{1}{3}\sqrt{5}\right) = \frac{2}{3} * \left(2\sqrt{2} + \sqrt{5}\right)$

d: Fläche eines Trapezes A = m*h = $\frac{a+c}{2} * h$

aus den bisherigen Ergebnissen: a = 2, c = $\frac{4}{3}$, h = $\frac{2}{3}$

also A = $\frac{2+\frac{4}{3}}{2} * \frac{2}{3} = \frac{\frac{10}{3}}{3} = \frac{10}{9}$