B.S. 164/1

a: $= (e^x)^2 - 2e^x e^{-x} + (e^{-x})^2 + 2e^0 = e^{2x} - 2e^{x-x} + e^{-2x} + 2e^0$

$$= e^{2x} - 2e^0 + e^{-2x} + 2e^0 = e^{2x} + e^{-2x}$$

b: $= 2e^x - \frac{e^1 * e^x}{e^1} - e^x * e^{2x} = 2e^x - e^x - e^{3x} = e^x - e^{2x}$

$$= e^x(1 - e^{2x}) = e^x(1 - e^x)(1 + e^x)$$

d: $= (\left(e^{\frac{1}{2}x}\right)^2 - 1)/(1 + e^{\frac{1}{2}x}) = ((e^{\frac{1}{2}x} - 1)(e^{\frac{1}{2}x} + 1))/\left(1 + e^{\frac{1}{2}x}\right) = e^{\frac{1}{2}x} - 1$

e: $= \left((e^4)^{\frac{1}{2}}\right)^x = e^{4*\left(\frac{1}{2}\right)*x} = e^{2x}$

B.S. 164/2

a: $f'(x) = 2x * e^e$

e: $f'(x) = 2 * (2 - e^x)(-e^x) = 2e^x(2 - e^x)$

i: $f'(x) = 2e^2 * \left(\frac{-e^{2x} * 2}{(4+e^{2x})^2}\right) = -4\frac{e^{2x+2}}{(4+e^{2x})^2}$

b: $f'(x) = 2e^x - 2e^{2x} = 2e^x(1 - e^x)$

f: $f'(x) = e^{1-0,5x^2} * (-x) = -x * e^{1-0,5x^2}$

j: $f'(x) = \frac{e^x * (x^2+1) - e^x * 2x}{(x^2+1)^2} = \frac{(x^2+1-2x) * e^x}{(x^2+1)^2}$

n: $f'(x) = 2e^{2x}$

B.S.164/3

I: $F'(x) = 4 * \frac{-e^x}{(1+e^x)^2} \rightarrow C$

II $F'(x) = 4 * e^{4x} \rightarrow D$

III $F'(x) = -e^{4-x} * (-1) = e^{4-x} \rightarrow A$

IV $F'(x) = \frac{1}{4} * \left[2 * (e^{2x} + e^{-2x}) * \left(e^{2x} * 2 + e^{-2x} * (-2)\right)\right]$

$$= (e^{2x} + e^{-2x})(e^{2x} - e^{-2x}) = e^{4x} - e^{-4x} \rightarrow B$$

B.S.164/4

a: =ln(1) = 0

b: $= [2]^2 + 2 * 1 = 4 + 2 = 6$

c: $= 2 + \ln(e^{-2}) - 3 * 3 + 2 = 2 + (-2) - 9 + 2 = -7$

d: $= \ln(3) + \ln(e) - 3 - \ln(3) = 1 - 3 = -2$

B.S.164/5

a: $\Rightarrow\ 2 * \ln(x) + 2 * \ln(x) = 1 \quad \Rightarrow \quad 4 * \ln(x) = 1 \ \left| * \frac{1}{4} \right. \Rightarrow \ln(x) = \frac{1}{4} \ \Big| \sim e$

$$\Rightarrow x = e^{\frac{1}{4}}$$

b: $\Rightarrow\ mit\ z = \ln(x) \quad Substitution$!!

$z^2 + 2z = 1 \Rightarrow\ z^2 + 2z - 1 = 0\ \Rightarrow z_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1}{2}\left(-2 \pm \sqrt{4 - 4 * 1 * (-1)}\right) = \frac{1}{2}\left(-2 \pm \sqrt{8}\right) = \frac{1}{2}\left(-2 \pm 2\sqrt{2}\right) = -1 \pm \sqrt{2}$

$$Resubstitution\ \ mit\ z_1 = \frac{1}{2}\left(-1 + \sqrt{2}\right)\ \ln(x) = -1 + \sqrt{2} | \sim e$$

$$x_1 = e^{-1+\sqrt{2}} \quad und\ analog{:}\ x_2 = e^{-1-\sqrt{2}}$$

c: $\Rightarrow \ln(x) + \ln(x+2) = \ln(e) - (\ln(e) - \ln(3)) = 1 - 1 + \ln(3) = \ln(3)$

$$\Rightarrow\ \ln\big(x(x+2)\big) = \ln(3)\ \ \Rightarrow x(x+2) = 3 \Rightarrow x^2 + 2x - 3 = 0$$

$$mit\ Vieta{:}\ \ x_1 = 1\ \ (\ oder\ x_2 = -3\ < 0\ )$$

d: Substitution z = ln(x)

$\Rightarrow (z-1)z = 2\ \Rightarrow\ z^2 - z - 2 = 0$

$$mit\ Vieta{:}\ \ z_1 = 2\ oder\ z_2 = -1$$

$$Resubstitution{:}$$

$1.Fall \qquad 2 = \ln(x) \mid \sim e \Rightarrow\ x = e^2 \quad 2.Fall \qquad -1 = \ln(x) | \sim e \quad \Rightarrow \quad x = e^{-1}$

e: Substitution: $e^x = z \quad e^{-x} = \frac{1}{e^x} = \frac{1}{z}$

$\Rightarrow \quad z + \frac{1}{z} = 2\ \mid * z$

$z^2 + 1 = 2z \Rightarrow z^2 - 2z + 1 = 0$

$mit\ binomischer\ Formel \qquad (z-1)^2 = 0\ \Rightarrow z = 1$

$Resubstitution\ de^x = 1\ \Rightarrow \quad x = 0$

f: Ein Produkt ist genau dann Null, wenn ein Faktor Null ist, d. h. aus dieser Gleichung lassen sich 2 neue Gleichungen ableiten:

I $\quad e^x - 2 = 0\ \Rightarrow\ e^x = 2 \mid \sim \ln \Rightarrow\ x = \ln(2)$

II $\quad$ x – 2 = 0 ⇒ $x = 2$